EXPLICIT RELATIONSHIPS FOR THE TERMINAL VELOCITY OF SPHERICAL PARTICLES

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The Turton–Levenspiel correlation for the drag coefficient of a sphere is employed to compare recently proposed explicit equations to predict the free-fall conditions. Predictions of four different expressions are explored over a wide range of Archimedes number.

A free-falling particle reaches its constant terminal velocity, when the weight of the particle is exactly balanced by the sum of the buoyancy and the resisting force caused by the flow of fluid around the particles. The force balance can be recast into the dimensionless form as

$$C_D Re_t^2 = \tfrac{4}{3} Ar \,. \qquad (1)$$

Clift et al.[1] presented a multi-segment correlation for the drag coefficient which consists of six polynomial equations with a total of 18 fitted constants. The need for such a complicated regression equation was questioned by Turton and Levenspiel[2]. These authors proposed the five-constant equation that correlates the available experimental data very well in the subcritical regime ($Re_t < 2 \,.\, 10^5$):

$$C_D = \frac{24}{Re_t}\left(1 + 0{\cdot}173 Re_t^{0.657}\right) + \frac{0{\cdot}413}{1 + 16\,300 Re_t^{-1.09}} \,. \qquad (2)$$

The correlation developed by Flemmer and Banks[3] also provides a very good description of experimental data on the drag coefficient of a settling sphere:

$$C_D = 24 \,.\, 10^E / Re_t \,, \qquad (3a)$$

where

$$E = 0{\cdot}261 Re_t^{0.369} - 0{\cdot}105 Re_t^{0.431} - \frac{0{\cdot}124}{1 + (\log_{10} Re_t)^2} \,, \qquad Re_t < 3 \,.\, 10^5 \,. \qquad (3b)$$

Our computations showed that the differences in predictions of Eqs (*2*) and (*3a,b*) range from -4% to $+8\%$. The mean value of the deviations amounts to $\pm 3{\cdot}5\%$. This compares well with an accuracy of $\pm 10\%$ (90% confidence interval) given by Turbon and Levenspiel[2] for their Eq. (*2*). Therefore, it appears that the free-fall conditions of spheres can be predicted by Eq. (*1*) in combination with Eq. (*2*) or with Eqs (*3a,b*) quite accurately throughout the entire Reynolds number range. It is of interest to note that the Eq. (*2*) and Eqs (*3*) are capable of showing a minimum value of the drag coefficient at a Reynolds number around 4 000.

In order to eliminate the need for an iterative solution, explicit relationships were recently developed to predict the terminal velocity. Zigrang and Sylvester[4] proposed an explicit equation for particle settling velocities in liquid–solid systems based upon the work of Barnea and Mizrahi[5] and Barnea and Mednick[6]. This general correlation was originally proposed to predict the fall velocity of the interface that develops during gravity sedimentations of monodisperse particles. In its limit this expression gives the terminal velocity of an isolated spherical particle. The original equation of Zigrang and Sylvester[4] can be rewritten into the dimensionless form as

$$Re_t = 1{\cdot}8329Ar^{1/2} + 29{\cdot}025 - (106{\cdot}4Ar^{1/2} + 842{\cdot}44)^{1/2} . \qquad (4)$$

Turton and Clark[7] and Wesselingh[8] developed empirical relationships of other form. Their expressions are weighted combinations of the asymptotic relationships for very low and very high Reynolds number. With the use of the Reynolds and Archimedes number as variables, these expressions can be recast into the form consistent with Eq. (*4*). The correlation of Turton and Clark[7] takes the form

$$Re_t = \frac{Ar^{1/3}}{(10{\cdot}82/Ar^{0.549} + 0{\cdot}6262/Ar^{0.137})^{1.214}} \qquad (5)$$

for $Re_t < 2 \,.\, 10^5$.

The interpolation formula of Wesselingh[8] also covers the entire region of Reynolds number and can be rewritten as

$$Re_t = \frac{Ar^{1/3}}{(13{\cdot}48/Ar^{0.6} + 0{\cdot}6074/Ar^{0.15})^{1.111}} \,. \qquad (6)$$

In a very recent work of ours[9], we have developed a polynomial equation for the free-fall conditions of a single particle:

$$Re_t = 10^{P(A)}R(A) \quad \text{for} \quad Ar < 10^8 \,, \qquad (7)$$

where

$$P(A) = ((0{\cdot}0017795A - 0{\cdot}0573)\,A + 1{\cdot}0315)\,A - 1{\cdot}26222 \qquad (7a)$$

$$R(A) = 0{\cdot}99947 + 0{\cdot}01853 \sin(1{\cdot}848A - 3{\cdot}14) \qquad (7b)$$

and

$$A = \log Ar\,.$$

The purpose of this brief communication is to compare the aforementioned explicit equations that make it possible to determine rapidly the particle terminal velocity.

RESULTS AND DISCUSSION

At first we made systematic computation of the free-fall conditions from Eqs (*1*) and (*2*) for selected values of the Archimedes number ranging from 1 to $4\,.\,10^7$. The equations were solved by a simple technique such as the interval halving with an accuracy of 5 significant figures. The computed values of the Reynolds number are presented in Table I. It is believed that these results are the best available representation of the free-fall conditions of a sphere.

The explicit expressions (*4*)–(*7*) can be viewed as more or less modified forms of a compilation of data on the drag coefficient of spheres. In order to compare the explicit equations, values of the drag coefficient were computed from the respective correlations with the aid of Eq. (*1*). Such results were compared to the values of the drag coefficient calculated with the use of the solutions presented in Table I. The relative deviations of the predictions of Eqs (*4*)–(*7*) are plotted in Fig. 1.

The equation by Zingrang and Sylvester shows a systematic error and the largest deviations. With respect to the primary purpose of the original correlation, this fact seems to be quite understandable. Both equations by Turton and Levenspiel and by Wesselingh show a similar behaviour. Their mean relative deviations amount to 11 and 10%, respectively. The maximum deviations do not exceed −22 and −20%, respectively. As can be seen in Fig. 1, Eq. (*7*) provides a very accurate description over the broad range of Archimedes number (i.e., $Ar < 4\,.\,10^7$). The average deviation amounts to 2·6% and the maximum deviation is 6·2%. The corresponding relative differences in Reynolds number can be estimated by Eq. (*8*)

$$\frac{\Delta Re_t}{Re_t} = \frac{1 - (1 + \Delta C_D/C_D)^{1/2}}{(1 + \Delta C_D/C_D)^{1/2}} \qquad (8)$$

that was deduced from Eq. (*1*). The maximum deviations of Eq. (*5*) (−22%), Eq. (*6*) (−20%) and Eq. (*7*) (6·2%) in C_D lead to maximum errors of less than 13%, 12% and −3%, respectively, in Re_t. These results are plotted in Fig. 2.

A useful extension of this work is to explore predicting the minimum fluidization from the free-fall conditions with the use of the empirical Richardson–Zaki relationship[10]

$$Re/Re_t = \varepsilon^n \tag{9}$$

Our experience suggests that the Richardson–Zaki exponent can reliably be predicted by the empirical correlation of Garside and Al-Dibouni[11] with an average deviation of less than 10%:

$$n = \frac{5{\cdot}09 + 0{\cdot}284 Re_t^{0.877}}{1 + 0{\cdot}104 Re_t^{0.877}}\,, \qquad 10^{-3} < Re_t < 3\,.\,10^4\,. \tag{10}$$

Using the voidage $\varepsilon_{mf} = 0{\cdot}4$, the minimum fluidization points were computed from Eqs (7), (9) and (10) and compared with the predictions of the Ergun equation[12,13]

$$\frac{1{\cdot}75}{\varepsilon_{mf}^3} Re_{mf}^2 + 150\,\frac{1 - \varepsilon_{mf}}{\varepsilon_{mf}^3} Re_{mf} - Ar = 0\,, \quad \psi = 1\,. \tag{11}$$

TABLE I

Free-fall conditions predicted with the use of the Turton–Levenspiel correlation for the drag coefficient[2]

Ar	Re_t	Ar	Re_t	Ar	Re_t
1	0·054178	500	14·023	90 000	454·52
2	0·10686	600	16·078	100 000	484·60
3	0·15849	700	18·028	200 000	733·40
4	0·20927	800	19·891	300 000	928·93
5	0·25930	900	21·680	400 000	1 095·2
6	0·30864	1 000	23·404	500 000	1 242·2
7	0·35743	2 000	38·312	600 000	1 375·1
8	0·40569	3 000	50·720	700 000	1 497·1
9	0·45335	4 000	61·701	800 000	1 610·5
10	0·50059	5 000	71·716	900 000	1 716·8
20	0·95173	6 000	81·013	1 000 000	1 817·0
30	1·3738	7 000	89·750	2 000 000	2 615·1
40	1·7747	8 000	98·030	3 000 000	3 213·6
50	2·1586	9 000	105·93	4 000 000	3 709·7
60	2·5287	10 000	113·50	5 000 000	4 141·1
70	2·8867	20 000	177·69	6 000 000	4 527·2
80	3·2344	30 000	229·91	7 000 000	4 879·3
90	3·5730	40 000	275·49	8 000 000	5 204·8
100	3·9033	50 000	316·61	9 000 000	5 508·6
200	6·8824	60 000	354·47	10 000 000	5 794·5
300	9·4788	70 000	389·77	$2\,.\,10^7$	8 062·9
400	11·836	80 000	423·01	$4\,.\,10^7$	11 199

The deviations range from -20% to 26% throughout the wide region of Archimedes number and they can be seen in Fig. 3. The extrapolation has been made from one fluidization limit to the other one. With respect to this fact, the agreement seems to be very reasonable.

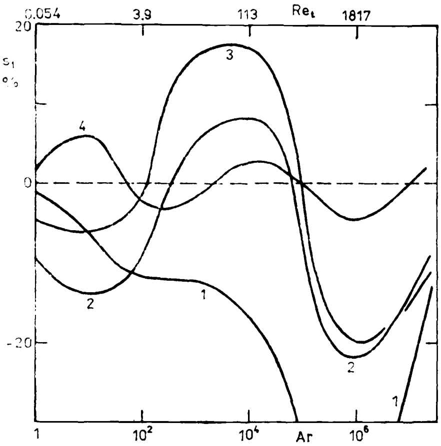

FIG. 1

Relative deviations of the drag coefficient determined from the explicit relationships for the particle terminal velocity. **1** Zigrang and Sylvester[4]; **2** Turton and Clark[7]; **3** Wesselingh[8]; **4** Hartman et al.[9]

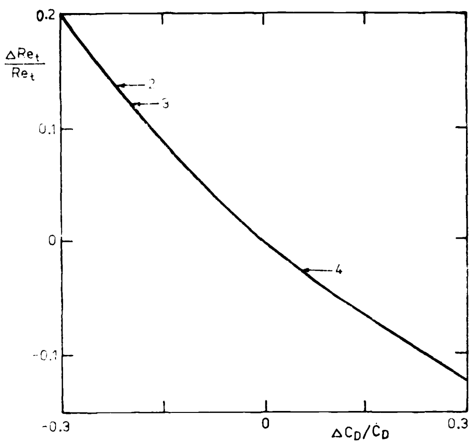

FIG. 2

Maximum errors of the explicit equations. $Ar \in \langle 1, 7 \cdot 5 \, . \, 10^7 \rangle$ **2** Turton and Clark[7]; **3** Wesselingh[8]; **4** Hartman et al.[9]. The solid line depicts Eq. (*8*)

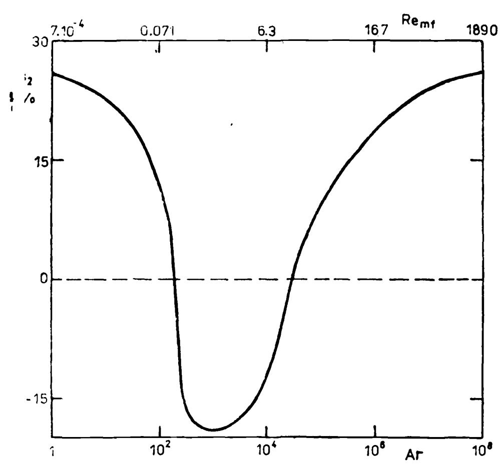

FIG. 3

Relative deviations of the minimum fluidization velocity determined with the use of the Richardson–Zaki equation[10] and the correlation of Garside and Al-Dibouni[11]. The Ergun equation[12] is taken as a standard

SYMBOLS

$Ar = d_p^3 g \varrho_f(\varrho_s - \varrho_f)/\mu_f^2$ Archimedes number
C_D drag coefficient of a sphere
ΔC_D inaccuracy in the drag coefficient
C_D^I drag coefficient determined from an explicit relationship for Re_t
C_D^{TL} drag coefficient predicted by the correlation of Turton and Levenspiel, Eq. (2)
d_p diameter of sphere, m
g acceleration due to gravity, m s^{-2}
n Richardson–Zaki exponent
$Re_{mf} = U_{mf} d_p \varrho_f/\mu_f$ Reynolds number at the minimum fluidization
Re_t Reynolds number at the terminal velocity of sphere
ΔRe_t inaccuracy in Re_t
Re_{mf}^E Reynolds number predicted by the Ergun equation, Eq. (*11*)
Re_{mf}^I Reynolds number determined with the use of the Richardson–Zaki equation, Eq. (*9*)
$s_1 = 100\,(C_D^{TL} - C_D^I)/C_D^{TL}$ relative deviation, %
$s_2 = 100\,(Re_{mf}^E - Re_{mf}^I)/Re_{mf}^E$ relative deviation, %
U_t free-fall (terminal) velocity, m s^{-1}
ε bed voidage
ε_{mf} bed voidage at the minimum fluidization
μ_f fluid viscosity, kg m^{-1} s^{-1}
ϱ_f fluid density, kg m^{-3}
ϱ_s particle density, kg m^{-3}
ψ particle sphericity

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